The Chapter on the Stars in an Early European Treatise on the Use of the Astrolabe (ca. AD 1000)

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The astrolabe was the most wide-spread astronomical instrument in the Middle Ages, both in the Islamic world and in Europe. The Europeans received the first knowledge of the instrument in the Christian area of North-Eastern Spain, around Barcelona. In the last two decades of the tenth century, about, they became acquainted with astrolabes and some texts related to the instrument from al-Andalus where at that time the leading astronomer was Maslama al-Majrīṭī (d. 398/1007-8). Writings on the astrolabe have only survived from several of his pupils, not from his own pen. But we have, in Arabic and in a (somehow confused) Latin translation, a table of 21 astrolabe stars set up for AD 978 by Maslama.[1] Subsequently Latin scholars in that area composed a number of treatises on

[1] Edited by P. Kunitzsch, *Typen von Sternverzeichnissen in astronomischen Handschriften des zehnten bis vierzehnten Jahrhunderts*, Wiesbaden, 1966, pp. 15-18, as "Type I", both the Arabic and the Latin versions. It should be added, however, that Maslama knew Ptolemy's *Planisphaerium*. He revised its Arabic text (which was then translated into Latin by Hermann of Carinthia, 1143) and added to it a number of notes and an extra-chapter as well as some chapters on the astrolabe. Maslama's notes were recently edited, in Arabic and in several Latin translations, by P. Kunitzsch and R. Lorch, *Maslama's Notes on Ptolemy's Planisphaerium and Related Texts*, Munich, 1994 (Sitzungsberichte, Bayerische Akademie der Wissenschaften, Phil.-hist. Klasse, 1994, 2). Here, in the Introduction, full information is given on editions and studies of the *Planisphaerium* and Maslama's contributions.

the description of the instrument, its construction and its uses. These writings may be called the "Old Corpus" on the astrolabe. The texts were edited by Millàs[2] and Bubnov[3]. One of these texts, *De mensura astrolabii*, inc. *Philosophi quorum sagaci* (called *h'* by Millàs), was accompanied by a table of 27 stars to be placed on the rete of the astrolabe. I have edited the table as "Type III" in 1966.[4] The oldest appearance of this table seems to be in MS Paris, BNF lat. 7412, fol. 5v. Here the table is still organized in the Arabic way: it has to be read from right to left (though a direct model of the table cannot be spotted in an Arabic source). It contains 26 stars (one star, η UMa, is omitted) with two coordinates: *mediatio coeli*[5] (here called *altitudo*) and another value (called *latitudo*) which may "best correspond to the maximum altitude of the stars at a geographical latitude of 39° (Valencia?)"[6]. The stars are mentioned with their Arabic names, in Latin transliteration; to several of the Arabic names tentative, often suitable, Latin translations are added.[7] In a re-organized form and with 27 stars, then, the table appears in the text *h'* and in endless repetitions in

2 J.M. Millàs Vallicrosa, *Assaig d'història de les idees físiques i matemàtiques a la Catalunya medieval*, Barcelona, 1931.

3 N. Bubnov (ed.), *Gerberti postea Silvestri II papae opera mathematica*, Berlin, 1899.

4 Kunitzsch, *Typen*, pp. 23-30.

5 I.e., the degree of the ecliptic culminating together with the star.

6 A proposal of E. Dekker; see P. Kunitzsch and E. Dekker, "The stars on the rete of the so-called «Carolingian astrolabe»", in *From Baghdad to Barcelona. Studies in the Islamic Exact Sciences in honour of Prof. Juan Vernet*, Barcelona, 1996, II, p. 656 n. 8. The table was edited, in an unsatisfactory form, by W. Bergmann, in *Francia* 8 (1980), 84; a partial edition was given by M. Destombes in *Archives internationales d'histoire des sciences* 15 (1962), 27 (Table II, columns 2-4).

7 See, e.g., *brachium* for *addirahan*, Arabic *muqaddam al-dhirāʿayn* (= Maslama's table, star no. 8; in the Latin version, star no. 16, also correctly *antecedens brachia*). This is a "ghost star", already in Maslama's table and then wandering through all versions of the star table of "Type III"; its Arabic name indicates α Gem, but its coordinates point to a region of the sky where α Hya would be the brightest star. Also on many Andalusī, Maghribī and Latin astrolabes the star, with this name, was subsequently inscribed in the approximate region of α Hya, according to the coordinates given in the star table.

manuscripts down to the 15th century. In this (main) form the table registers as *latitudo* the mediations, and as *altitudo* a special value never mentioned in Arabic texts and in later Latin astrolabe treatises; the value seems to be read off from an (Arabic) astrolabe in the hands of the first author of that type of the table. The *mediatio* (here: *latitudo*) could be read off on the graduated ecliptic ring of the rete of an astrolabe by putting a ruler through the centre of the instrument and through the star. To find the second coordinate, here called *altitudo*, the ruler was put at right angles through the first line at the position of the star until the graduated outer rim of the astrolabe. Here, the number of degrees between the meeting point of the first line on the rim and each of the two meeting points of the second line were counted, and this (i.e., in one of the two directions) is the value called *altitudo*.[8] In this form the table was taken over by Ascelinus of Augsburg (early 11th c.) in his treatise on the construction of the astrolabe,[9] and afterwards - obviously from Ascelinus - by Hermannus Contractus (1013-1054), also in a treatise on the construction of the instrument.[10]

Somewhere in time between the "Old Corpus" and Ascelinus may be located a text on the description and the uses of the astrolabe, commonly known as *De utilitatibus astrolabii*, Inc. *Quicumque astronomicae discere peritiam disciplinae*, edited by Bubnov[11] and called by him "J". Its author is unknown. Some scholars ascribed it - with little probability - to Gerbert of Aurillac (ca. 940/50-1003; as pope, Silvester II, 999-1003); Bubnov

8 In some later reproduction of this star table, this coordinate was aptly called *longitudo ex utraque parte* ("Type XI", sources *a-d*, in Kunitzsch, *Typen*, pp. 67-71). The numerical values of this coordinate are different from those of the *latitudo* in MS 7412.

9 Edited from only one, incomplete, manuscript by W. Bergmann, *Innovationen im Quadrivium des 10. und 11. Jahrhunderts*, Stuttgart, 1985, pp. 223-225; new edition, from five manuscripts (with additional notes from a sixth manuscript on a separate sheet added to the paper), by C. Burnett, in *Annals of Science* 55 (1998), 343 ff., see also Figs. 13-15.

10 Edited, from MS Munich, Clm 14836 (written still during Hermannus' life-time), by J. Drecker, in *Isis* 16 (1931), 200-219.

11 Bubnov, *Gerberti opera mathematica*, pp. 109-147.

edited it as a dubious work of Gerbert; others ascribed it, more cautiously, to a pupil of Gerbert. Chapter 17 of the treatise is entitled *De vocabulis Latinis et Arabicis stellarum et formationibus earundem*. In it the author tries to identify the stars of the table of "Type III", which were registered there only with their Arabic names, among the traditional classical constellations that were transmitted in Latin texts since late Antiquity.

The transmission of the text *J* is in itself utterly complicated. Bubnov edited it as a text of altogether 21 chapters. Some years ago, W. Bergmann[12] tried to demonstrate that there exist, in the numerous manuscripts, two versions of the treatise: the original one, in 19 chapters, and a version revised by Hermannus Contractus, in 21 chapters. However, afterwards it was observed[13] that this straightforward classification obviously cannot be maintained any more since, e.g., one manuscript (Munich, Clm 560) claimed by Bergmann for the basic 19-chapter version, does contain the 21 chapters and some other elements noted by Bergmann for the 21-chapter version and was copied in the early 11th century, i.e. before the time of Hermannus' activity. (Lately I inspected MS Clm 560 *in loco* and found these observations fully confirmed.) I shall not enter here into more details about this delicate problem, especially because I do not have copies from all the relevant manuscripts at hands. Moreover, I shall discuss a few selected items from the star descriptions in ch. 17 of the treatise which are in themselves doubtful.

The description begins in the north, with Benenaz = η UMa (to which some manuscripts add its second Arabic name, Alkaid, sometimes written as Alcaio), and proceeds towards the south, with some irregularities.

[12] Bergmann, *Innovationen*.

[13] Private communications of Prof. A. Borst, Konstanz, who has collected and collated all these manuscripts in connection with preparations for a new critical edition of Hermannus' scientific writings. Important in this respect is also the "Konstanz Fragment", four pages from a manuscript written at Reichenau around AD 1008 and containing the latter part of our chapter 17; see the edition by A. Borst, *Astrolab und Klosterreform an der Jahrtausendwende*, Heidelberg, 1989 (Sitzungsberichte der Heidelberger Akademie der Wissenschaften, Phil.-hist. Klasse, 1989, 1), esp. pp. 112-127. This very early fragment contains already elements ascribed by Bergmann to the revised version of *J*.

Several of the Arabic star names - which are written in all the sources in a great variety of misspellings - are confused. For Alrif (α Cyg) an alternative form Archeitus, Arrectus etc. is mentioned[14], the derivation or background for which is not obvious.

In Auriga two names are mentioned, *Menreb Alroech id est humerus* (of this, *Menreb id est humerus* belongs to Orion, not to Auriga; for Alroech a better spelling in some manuscripts is Alhaioch which is the correct Latinized Arabic name of α Aur), and *Rigel id est pes* (this is the correct name for β Ori and belongs there; it is as such correctly mentioned below, *in Geminis*, in the sign of Gemini).

The confusion about Telum, Aquila and Alhadib has been settled by Bergmann.[15] Apparently the intention was to assign Alhadib[16] to Telum (*sic* instead of Cassiopeia), and Alceir (α Aql) to Aquila (where some manuscripts, instead, refer to Cygnus a second time).

In the sign of Gemini two stars of Orion are mentioned: *Alhaioch Algeuze id est humerus* (α Ori; here wrongly the word Alhaioch - name for α Aur - is written, whereas the element Menreb belonging to α Ori was wrongly added above to the name of α Aur) and *Rigel id est pes* (β Ori; Rigel was wrongly mentioned above under Auriga also, and we shall find it a third time later, below).

In the sign of Leo two stars are listed: *Aldiraan id est frons*, and Calbalazeda (α Leo). For Aldiraan, a "ghost star", cf. note 7, above. Neither the Arabic name nor the coordinates (in the table) contain an

[14] See Bubnov, *Gerberti opera mathematica*, p. 137, 3. This star's description is omitted in Bergmann's parallel edition of the chapter in the two versions: Bergmann, *Innovationen*, p. 221.

[15] Bergmann, *Innovationen*, p. 77f.

[16] This star appears already in Maslama's star table (Kunitzsch, *Typen*, pp. 15-18, star no. 21, with the name of β Cas, but with confused coordinates - longitude and latitude alone, mediation and declination not given - rather pointing to the area of Pegasus). Also in the star table of "Type III" the coordinates of Alhadib are odd; in MS 7412, 5v the mediation is given as *Trutina* [leg. *Truta* = Pisces!] XVI°, which appears in the other versions as Taurus [*sic*] XXII°. Astronomers really trying to locate this star in the sky were of course perplexed.

allusion to *frons*, i.e. the Lion's forehead. It is obvious that here some reader, not content to be unable to find the identity of Aldiraan, tried to identify this star with some star in Leo; he chose *frons*, a name occurring in other texts belonging to the same period as the "Old Corpus" of astrolabe treatises, as the name of the 10th Lunar Mansion, *al-jabha*, "the (Lion's) Forehead", consisting of $\zeta\gamma\eta\alpha$ Leo. Since α Leo appears among the astrolabe stars separately, this reader probably aimed at ζ Leo. This interpolation appears in two forms: often *id est frons* is simply added to the unintelligible name Aldiraan; but in some manuscripts, instead of Aldiraan the true Arabic name of *frons* is found: Aliebaha (MS Munich, Clm 14763) or Algebaha (MS Ripoll 225[17]).[18]

After completing the stars in the zodiac the author arrives at the stars of the southern hemisphere. Here he seems to be confused, unable to identify the names with certain stars (though in the area of northern France, where these identifications were probably made, all of the astrolabe stars mentioned in the table of 27 stars could well be observed). He mentions ι and ζ Ceti, *quae aut raro aut numquam in nostris climatibus cernuntur*. Hereafter he continues: *Est et Rigel et Alhabor*. The second of these two stars is Sirius, α CMa, the brightest of the fixed stars, which must have been nicely visible in the author's place. The first star is enigmatic.

In manuscripts of the 21-chapter version it is called Rigel, i.e. with the name of β Ori mentioned already above, once in its correct place and once erroneously under Auriga. In manuscripts of the 19-chapter version -but also in MS Munich, Clm 560, and in the Konstanz Fragment (note 13) - we find, instead, the name Addeleni. Bergmann assumes that the revisor of J - in his opinion, Hermannus - has replaced the unintelligible Addeleni by Rigel (though this makes not much sense, because Rigel was already mentioned earlier, in its correct place, as β Ori). In its earliest form, it seems, the name was spelled Addelem (Paris, BNF lat. 7412, 8r; Munich,

17 *Apud* Millàs, *Assaig*, p. 155 note 1.

18 Cf. the Lunar Mansion name *Alcebata - frons*, from the *Mathematica Alhandrei summi astrologi*, MS Paris, BNF 17868 (10th c.), in Millàs, *Assaig*, p. 251.

Clm 560;[19] Leiden, Scal. 38). This then degenerated into other forms, such as Addeleni, Adelem and Addelen. Until the moment of writing these lines I am not able to explain this name (just as Archeitus/Arrectus, above). It does not echo any of the Arabic names of the 27 stars contained in the list of astrolabe stars accompanying the "Old Corpus" of astrolabe texts. One would not easily be ready to assume behind this word the name of any other star, outside the 27 stars of the tradition.

The text continues with three other names of stars on which the author "has nothing to say" (*Quid autem dicam de his dum a nobis minime videntur*?): Algomeiza Aldirnam (the first word is the name of α CMi; the second is wrongly repeated from above, under Leo); Ganamalgurab (γ Crv) and *Alcasal*[20] *vel Alhimech* (α Vir),[21] both of which are located far too deep in the south, *in Centauro*.

Four stars out of the 27 are totally omitted in the description: α Oph (Alhauui), β Per (Algol), θ UMa (Arrucaba) and ι UMa (Egreget).

In the present state of exploration of the history and transmission of the text J, I think, it would be premature to give definite explanations of how, when and by whom the various corruptions and deficiencies were brought into the text of this chapter. At least, most of the Arabic star names here mentioned can be traced in the tradition of the table of 27 astrolabe stars

19 Bubnov (*Gerberti opera mathematica*, p. 138, App. under *az*) has wrongly registered from MS Mon H (= Clm 560) the spelling Addeiemz. This spelling caused me, in my book *Arabische Sternnamen in Europa*, Wiesbaden, 1959, p. 70f., no. 19, to explain Addeiemz (et varr.) as a corruption of the Arabic *al-jawzā'*, the name of Orion and also Gemini (in Latin, *alieuze* → *alienze* → *addeiemz*). After inspecting MS Clm 560 *in loco*, this explanation must be withdrawn. The true reading of the entire phrase in Clm 560 is: *Est et AdDELEMZALHABOR*. The copyist wrote Addelem; then he mistook the abbreviation "et" between the two Arabic names as a Z and wrote the two separate names together in one conglomeration. So we know that Clm 560 has the spelling Addelem and that this name cannot be derived from Arabic *al-jawzā'*, Orion.

20 Clm 560 has Algazal, not registered by Bubnov, *Gerberti opera mathematica*, p. 138, App. under *bg*.

21 The two words together render the full Arabic name of α Vir, *al-simāk al-aʿzal*, "the unarmed Simāk"; but the element Alcasal/Algazal is only rarely found in Latin in the "Old Corpus". The star table in BNF 7412, 5v, has: Alhazel.

("Type III"), except for Archeitus/Arrectus and Addelem/Addeleni for which no plausible explanation is ready at hand.

On the occasion of this paper I should like to mention another unexplained star name belonging to the same tradition. MS Paris, BNF 7412, contains, after a compilation of portions of texts from the "Old Corpus", on foll. 19v-23v drawings of the rete, the seven plates (constructed for the latitudes of the seven climates) and the back of an Andalusī astrolabe. Its inscriptions, in Arabic, in the Andalusī cufi ductus, were also copied by the Latin draftsman. On the back, he even copied the maker's name: Khalaf ibn al-Muᶜādh.[22] While the inscriptions of all the other parts of the instrument are copied in Arabic characters, the names of the 27 stars on the rete on fol. 19v are given in Latin transliteration. To each name in the rete a serial letter or mark is added, and at the bottom of the page for each of these letters or marks alternative spellings of the respective names are given. All the names here mentioned belong to the tradition of the 27 stars of "Type III".[23] Against Addiraan, in the rete, we read, written upside down: *vel liragenz* (or *-geni*). If the reading *-genz* is correct, one would assume here another corruption of Arabic *al-jawzā'*, Orion or Gemini. With the reading *-geni*, no explanation comes up. The first part of the word, *lira-*, remains also unexplained. For a better understanding other spellings of the name in other manuscripts must be awaited.

[22] Cf. the description by P. Kunitzsch, "Traces of a tenth-century Spanish-Arabic astrolabe", in *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 12 (1998), 113-120.

[23] These names were edited by Kunitzsch, *Sternnamen*, p. 90f. After having available a better photograph from the manuscript, I here can correct a few readings: at *a*, column 2, read: Alwagakba; at *b*, read Alkaio; at *l*, in the Arabic, read *munīr al-fakka*.

Some Early Applications of the Sine Quadrant

Richard Lorch

The sine quadrant is an instrument in the form of a graduated circular quadrant with lines drawn parallel to one of the straight sides (see Figure 1, which has lines for every ten degrees)[1]. In addition, there is a rigid rule pivoted at the centre of the quadrant (*A* in Figure 1) or a piece of thread one end of which is fixed there. On the rule temporary marks may be made or else it carries a permanent scale; on the thread there is a bead. In each case radial distances from A may be temporarily preserved. Sine quadrants were made either as separate instruments or as one or more of the quarters of the back of the astrolabe. In the latter case the alidade served as the rule. The sine quadrant may also take the form of a graphical procedure in which the quadrant and rule are drawn and the function of the bead or mark is taken over by a pair of compasses or similar device.

Multiplication and division of sines may be carried out with the sine quadrant by using the relation $x = r \sin \theta$ in Figure 2, together with $x = \sin \psi$ (if we suppose the radius $AB = 1$) and a similar relation for r (which $= AE'$). Here *AD* represents the rule or string, *E* the mark or bead, and *ZEH* one of the parallel lines on the instrument. Multiplication by the sine of an angle and division by the cosine of the same angle can be performed

1 Sometimes additional lines were drawn, e.g. radial lines, quadrants concentric with the outer quadrant, or a semicircle whose diameter is one of the straight sides. See Schmalzl [1929], pp. 62-99, on "Dastūrquadranten" and "Sinusquadranten", which he says (p. 83) are closely related. We are here concerned only with the most primitive type.

simultaneously, for, in Figure 3, $\sin\psi = (\sin\phi \,/\, \cos\phi)\sin\theta$. Theoretically, the sine-quadrant could also be used to solve this equation for ϕ if it had lines parallel to both straight sides.

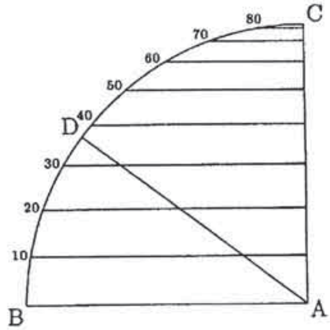

Figure 1

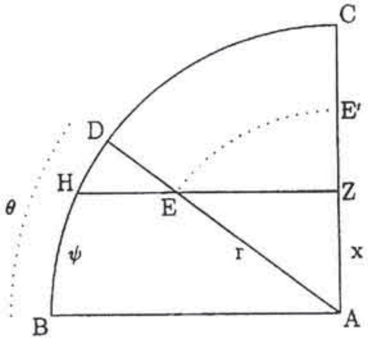

Figure 2

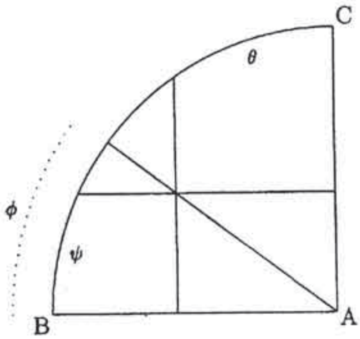

Figure 3

There are few early texts or surviving instruments. Latin texts edited by Millás on a form of the sine quadrant for finding the time in seasonal hours[2] are of the late tenth century. Also of the tenth century are two astrolabes having sine quadrants on the back[3]. In the same codex as treatises on the astrolabe by al-Farghānī and al-Khwārizmī there is a short, anonymous treatise on the sine quadrant, which appears to be of early date and which may quite probably be by al-Khwārizmī[4]. Another text, on the horary quadrant (which was probably developed from the sine quadrant[5]), may also be attributable to al-Khwārizmī[6]. Thus the sine quadrant seems to be at least as old as the ninth century.

Of the few texts on the sine quadrant in the earlier period of Islamic astronomy the two presented here are arguably of the tenth century or older. The manuscript in which they are found, Paris BN ar. 2457, appears to be in one hand, except for the last few folios, and several of the texts it contains have colophons bearing dates between 358H (969 AD) and 340 of the Yazdajird era (972 AD) and giving the name of the scribe as Aḥmad ibn Muḥammad ibn ᶜAbd al-Jalīl, i.e. the celebrated mathematician al-Sijzī[7]. It has recently been argued that Sijzī cannot have written the codex: the numerous mistakes in the texts cannot have been made by a mathematician of Sijzī's calibre; and in the last sentence of a work by Sijzī himself the scribe refers to him in the third person, "This is the end of what he wrote of this book". So the whole is probably a later copy of

[2] Millás [1932], pp. 225 (repr. 73) ff.

[3] See Girke and King [1988], p. 47 and Plate 9a for an astrolabe dated 959-960 (with vertical as well as horizontal lines), and p. 51 and Plate 9g for another dated 984. The latter has vertical lines and concentric quadrants as well as horizontal lines.

[4] King [1983], p. 29. The text on the sine quadrant is in MS Berlin, Landberg 56, ff. 96v-97v, in the part of the codex catalogued by Ahlwardt as No. 5793.

[5] Lorch [1981].

[6] King [1983], p. 31. The manuscript is Istanbul, Aya Sofya 4830, ff. 196v-197r.

[7] The chronological ordering of the colophons, the various foliations of the codex and the ordering of the items is considered in Kunitzsch and Lorch [1993].

Sijzī's codex[8]. To the first objection one could counter that copies made by anyone in a hurry will contain mistakes. In this context we may note that the misplaced diagrams in the treatise on the "melon-shaped" astrolabe, ff. 141r-149v, later had their correct places indicated[9]. As for the second objection it could be said that the last sentence is near enough in style to a colophon to excuse the use of the third person. At all events, the *ductus* of the script is appropriate to the tenth century and is almost certainly no later than the eleventh[10].

In the *Maqālīd* al-Bīrūnī mentions Sijzī's predilection for collecting rules of calculation, taken from the best geometers and writers of *zīj*es, but without proofs. In a note to this passage M.-Th. Debarnot points to the short text in MS BN ar. 2457, f. 56r, as an example[11]. Since the first of our passages (on f. 93r, item 21 in De Slane's catalogue) is in this category, it is indeed likely that the various items were collected by Sijzī and that he had found them in *zīj*es and similar works.

The second passage (f. 95r, item 23 of De Slane's catalogue) is one of several short texts following the collection on f. 93r and preceding Thābit ibn Qurra's treatise on the measure of paraboloids. Interesting enough, the piece immediately following (on the position of the Moon) mentions the *Sindhind*, but this has no bearing on our text.

In the text on f. 95r the elementary operations are relatively straightforward: arcs are measured from the 90-point (according to the interpretation suggested here) of the altitude scale on the back of an astrolabe; versed sines are found from arcs and *vice-versa*; and distances are transferred from one place to another with the alidade. We may assume that the quadrant was marked with degrees as usual and that the alidade

8 Hogendijk [1996], p. 8.

9 See Kennedy, Kunitzsch and Lorch [1999], p. 13

10 Prof. P. Kunitzsch, private communication.

11 Bīrūnī [1985], p. 96. In n.1 Dr. Debarnot identifies the rules given on f. 56r and notes that at least two out of the three rules given there are of Indian origin. She points out that the date of the manuscript is not certain for this text, which could have been added later. The same goes for the two texts given here.

was supplied with a uniform scale.

In the first text the fourth and the last procedures do not belong with the rest. The fourth is simply a rule of calculation without the use of the sine quadrant in any form. By contrast, the last item refers to a material instrument. We may assume that there were horizontal lines and that the alidade was non-uniformly graduated, so that a graduation represented not the distance from the centre of the instrument, but the arc whose sine this distance represents.

Of the other items, [1], [2], [7] and possibly [6] embody straightforward multiplication or division of sines. In [3] and [5] there is also a simultaneous multiplication by a sine and division by the cosine of the same angle. Strictly speaking, items [1] - [3] and [5] - [7] are rather graphic solutions to problems in spherical astronomy, but the construction is limited to a quadrant and the operations are precisely those of a sine quadrant. Further, the problem-solver must have a ruler available that is non-uniformly graduated (see the end of the last paragraph), i.e. a graduated alidade. This alidade is implicit in items [2], [3], [5] and [7] and is the more remarkable since it is easy to replace the use of such a graduation by copying the distance on the ruler onto one of the sides of the quadrant.

There appears to be some relation to graphical procedures by other authors, e.g. the ninth-century astronomers Ḥabash al-Ḥāsib, in his treatise on the melon-shaped astrolabe[12] and al-Māhānī, in a text beginning "Maᶜrifat al-samt li-ayy sāᶜa ..."[13]. These differ from the sine-quadrant procedures by operating in a basic circle rather than a basic quadrant and in having no non-uniformly graduated rule, but many of the operations are the same. Some of these procedures are analemmas - i.e. they are geometrically conceived in their entirety, involving orthogonal projections and the folding down of one plane on another, etc. - or may be interpreted

[12] See Kennedy, Kunitzsch and Lorch [1999], pp. 18-63, 91-121.

[13] *Ibid.*, 149-159, 169-172.

as such[14]. Some of Ḥabash's and Māhānī's procedures, however, appear not to be analemmas[15] . Rosenfeld has recently coined the term "geometric trigonometry" for such geometrical representation of trigonometrical calculations, on analogy with the "geometric algebra" of the second book of Euclid's *Elements*[16]. His first example, from al-Khwārizmī (9c.), is equivalent to our item [7], but there is no non-uniformly graduated rule. Suffice it to say here that with the sine-quadrant procedures and with some of Ḥabash's and Māhānī's graphical solutions an essentially trigonometrical, or arithmetic, element is interposed between the three-dimensional problem and the two-dimensional solution. We may note that it is sometimes hard to guess what the history of a particular procedure was. For instance, the formidably trigonometric-looking procedure at the beginning of Māhānī's treatise has recently been shown by Kennedy to be justifiable by an analemma[17]: how the solution was conceived we do not know.

Since the texts edited here[18] both show signs of being battered about, they are introduced almost without change. The diagrams on f. 93r have been redrawn as closely as possible to the originals. For convenience they (Figures 4, 6, 7, 8 and 10) are placed next to the summaries in the commentary. Items [4] and [8] need no lettered diagrams. The diagram for [6] is missing. Dotted lines have been added.

[14] See Berggren [1991-1992] for an interpretation by analemma of one of Ḥabash's procedures.

[15] E.g. Ḥabash's "Problem 2". See Kennedy, Kunitzsch and Lorch [1999], 26-37 and 98-103. Similarly "Problem 4": see *ibid.* pp. 107-111 and Lorch [1998], 222-223.

[16] Rosenfeld [1993].

[17] See Kennedy, Kunitzsch and Lorch [1999], p. 170-171. The procedure was first studied by Luckey [1941], p. 200 and fig. M.

[18] I am most grateful to Prof. Paul Kunitzsch for help in transcribing and translating these texts.

Text I (*BN ar. 2457, f. 93r*)

[1] إذا كان ا جـ عرض البلد وأخرج د جـ و ه ب عدد السمت المطلوب عمله نخرج ه ز موازيا لـــ د ب ونجعل د ز مثل د ح ونخرج ح ط موازيا لـــ د ز فإن عدد ط ب إذا ألقينا <ه> من المقنطرات من مدار الحمل على تقاطعه وخط وسط السماء في الجهتين وأدرنا على مركز الصفيحة وببعد ذلك العدد وعلّمنا على الأفق في الجهتين علامتين فنقطة ص ونقطة العلامة هما مجاز لدائرة ذلك السمت فنطلب مركزها على خط مراكز السموت وذلك سهل إن شاء الله.

[2] إذا كان ا د تسعين وكان ب د مقدار عرض البلد لهذه الصفيحة وا ز قوس مقدار السمت التي نريد مثلا عشرة و ه ز يوازي ا جـ فإن قوس ه جـ محصل فنأخذ بالبركار بعد ه جـ ونضع أحد رأسي البركار على تقاطع مدار الحمل وخط وسط السماء والآخر حيث وقع الرأس الآخر[19] من جهة المركز ثم ندير على مركز القطب وببعد الذي وقع الرأس الآخر للبركار[20] دائرة تقطع الأفق فالموضع القاطع من الأفق <...> فإن سمت العشرة وقع على تقاطعه <...> الأفق وعلى نقطة ص أعني سمت الرأس ومركز دائرته يقع على خط مراكز السموت.

[3] لمعرفة مطالع البروج في الفلك المستقيم،
إذا كان ا د ميل أول الثور وأخرجنا د ه موازيا لـــ ا ب و ا ز تمام الميل الأعظم ونخرج ب ز ونخرج من تقاطعه د ه وهي نقطة ط خط ط ح يوازي ب جـ ونخرج ب د يقطع ط ح على ك فإذا كان ب د مقسوما بـــص يكون ب ك مطالع أول الثور.

[4] إذا ضربنا جيب ارتفاع الوقت في س وقسمناه على جيب ارتفاع نصف النهار وقوسناه وأخذنا لكل يه ساعة تخرج لنا ساعات معوجات.

[19] الآخر: فوق السطر.

[20] للبركار: البركار في م.

[5] إذا كان ا ط تمام الميل ونقطة ه تقاطع ب د و ي ط[21] إذا كان جـ د عرض البلد و ب ط مقسوم بـ ص يكون ب ح نصف زيادة النهار الأطول وكذلك في سائر البو...[22].

[6] إذا كان قوس ا د عرض البلد وقوس جـ ح تعديل النهار و ب ه جيبه ونخرج ب د ونجعل ب ز مثل ب ه ونخرج ز ط يوازي ه ح فيكون ج ط جملة الميل[23].

[7] إذا كان ب جـ بعد الكوكب عن مدار الحمل وذلك سهل المأخذ وكان ب د ارتفاع رأس الحمل بالبلد وكان ا د مقسوما بتسعين فإن ا ز يكون سعة مشرق الكوكب في الجهة التي هو فيها.

[8] معرفة سعة مشرق أي درجة شئت بأي عرض تريد فألق عرض البلد من تسعين درجة فما بقي فضع مري العضادة على تلك الدرجة من أجزاء الارتفاع ثم اجعل الدرجة إلتي تريد سعة مشرقه ميلا وانظر إلى أجزاء الارتفاع بمقدار درجات الميل وخذ ما بحياله من الأجزاء التي على ظهر العضادة الأجزاء الستين على كم جزء يقع فما كان فهو سعة المشرق.

[1] When *AG* is the latitude of the region, *DG* is drawn, *EB* is the amount [*ᶜadad*] of the azimuth it is required to construct, we draw *EZ* parallel to *DB*, make *DZ* the equal of *DH* and draw *HT* parallel to *DZ*. When we take away the amount *TB* from the almucantars [starting] from the day-circle of Aries at the intersection of it and the meridian-line on the two sides, and about the centre of the plate and with the distance of that amount [i.e. from the centre to one of the places on the meridian just found] we draw [a circle], making marks on the horizon on the two sides, then the 90-point and the point of the mark [i.e. one of the two marks just found on the horizon] are the passage for the circle of that azimuth. Its centre is sought

[21] ي ط: ا ط في م.

[22] غير مقروء

[23] كُتب أوّلا "الجيب" ثمّ شُطب وكُتب بدلا منه "الميل"

on the line of centres of the azimuths; and that is easy, God willing.

[2] When *AD* is ninety, *BD* is the quantity of the latitude of the region for this plate, *AZ* is the arc of the quantity of the azimuth that we want - for example ten [degrees] - and *EZ* is parallel to *AG*, then arc *EG* is the resultant [*muḥaṣṣal*]. So with the compasses we take the distance *EG*, placing one head of the compasses on the intersection of the day-circle of Aries and the meridian line and the other wherever the other head falls on the side of the centre. Then about the centre of the pole [*sc*. about the pole as centre] and with the distance [at] which the other head of the compasses fell we draw a circle cutting the horizon, and so the cutting-place of the horizon [...] . The azimuth of ten [degrees] falls on its intersection [with] the horizon and on the 90-point, i.e. the zenith; and the centre of its circle falls on the line of centres of the azimuths.

[3] *To determine the ascensions of the signs at* sphaera recta.
When *AD* is the declination of the beginning of Taurus, and we draw *DE* parallel to *AB*, and *AZ* is the complement of the maximum declination, and we draw *BZ*, and we draw from its intersection [with] *DE*, which is point *T*, line *TH* parallel to *BG* and draw *BD* cutting *TH* at *K*, and when *BD* is divided into 90, [then] *BK* is the ascension of the beginning of Taurus.

[4] When we multiply the sine of the altitude of the time by 60, divide it by the sine of the altitude at midday, make it [i.e. the resulting sine] an arc and for every 15 [degrees] we take an hour, there emerges for us curved [i.e. unequal] hours.

[5] When *AT* is the complement of the declination, point *E* is the intersection of *BD* and *YT* - when *GD* is the latitude of the region - and *BT* is divided into 90, then *BH* is half the excess of the longest day. Similarly for the rest of the ... .

[6] When arc *AD* is the latitude of the region, arc *GH* is the equation of the day and *BE* its sine, and we draw *BD*, make *BZ* the equal of *BE* and draw *ZT* parallel to *EH*, then *GT* is the totality of the declination.

[7] When *BG* is the distance of the star from the day-circle of [the beginning of] Aries - and that is easy to obtain -, *BD* is the altitude of [the day-circle of] the beginning of Aries in the region and *AD* is divided into ninety, then *AZ* is the rising amplitude of the star on the side that it is on.

[8] *Determination of the rising amplitude of any degree you wish in any latitude you want.*
Subtract the latitude of the region from ninety degrees. Whatever remains, put the pointer of the alidade on that degree of the altitude degrees. Then make the degree of which you want the rising amplitude a declination. Look at the degrees of altitude in the quantity of degrees of the declination. Take what is opposite it of the degrees that are on the back of the alidade, the sixty degrees [*ajzā'*], at which degree it falls. What it is is the rising amplitude.

Commentary and Mathematical Summary

In the following R is the radius of the quadrant, Sin $\eta = R \sin \eta$, Cos $\eta = R \cos \eta$. δ is the declination, ϵ the maximum declination, α the ascension, ½*eq* half the equation of day, ϕ the latitude of the region, *r.a.* the rising amplitude, *az* the azimuth of some circle through the zenith; a parallel-circle is a circle parallel to the equator. A bar over a quantity means its complement.

[1] The purpose of the first two procedures is, in laying out an astrolabe, to map one of the feet of a selected azimuth on the horizon by finding the declination of the point. Together with the image of the zenith (the "ninety-point"), this will give the centre of the azimuth, since this centre also lies on the easily constructible line-of-centres perpendicular to the meridian.

Procedure

$AG = \phi$
draw DG
$EB = az$
$EZ \parallel DB$
$DZ = DH$
$HT \parallel DZ$ [*read DB*]
BT [$= \delta$]

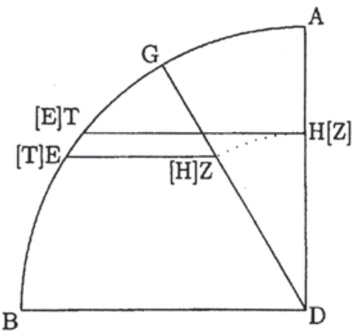

Figure 4

Figure 4 with unbracketed letters represents the diagram in the manuscript. Now Sin $BT = DH = DZ =$ Sin $az \times R$ / Cos ϕ. If the diagram is accepted with the letters emended as in the square brackets, then Sin $BT =$ Sin az Cos ϕ / R , i.e. $BT = \delta$. The only remaining problem is the line "make DZ the equal of DH", as if DZ were being defined, but this is a problem under any explanation.

Having found δ, the author finds the corresponding parallel-circle on the astrolabe by counting off the same amount on the meridian among the almucantars, starting with the intersection with the equator (i.e. from point A in Figure 5), to reach Z. The almucantars provide a convenient measure for the projected declination, since before projection they are evenly spaced on the meridian. With centre E he draws a circle through Z and finds the intersection H with the horizon.

We note that az must be measured from the east or west point. Each positive (i.e. northern) δ will produce two points on the horizon (in Figure 5 H and T) and so two azimuth circles - those with azimuth az from the east or west point. Two further azimuths are found with the corresponding negative (or southern) δ.

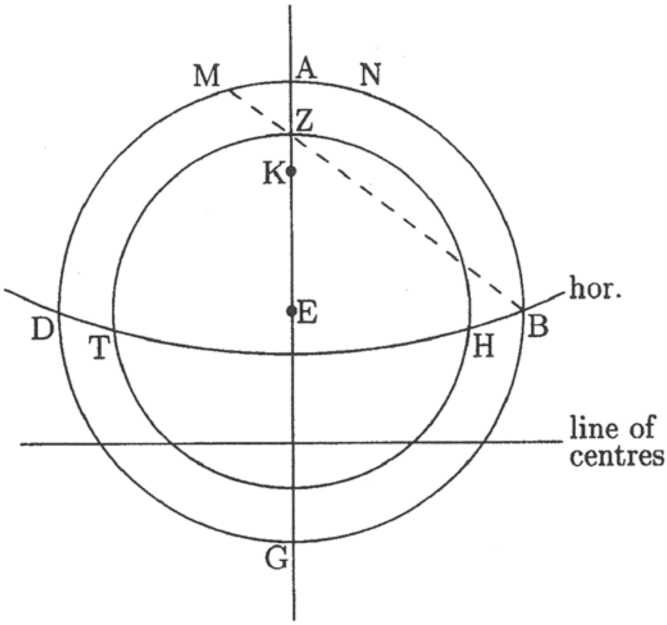

Figure 5

[2] In this procedure there is a different way of transferring the declination to the drawing of the astrolabe plate. As it stands, the text says that the distance *EG* (arc *EG* is the declination) is transferred to the astrolabe diagram by making *AZ* (in Figure 5) equal to the chord of *EG* (of Figure 6). But what is required is not Chord(*EG*), i.e. $2\mathrm{Sin}\frac{1}{2}\delta$, but $1\text{-}\tan\frac{1}{2}\bar{\delta}$. There is a perfectly good method of transferring the declination by means of its chord: in Figure 5 put AM = Chord(δ) and join *M* to *B* (or *D*); the intersection of *MB* and *AG* will be the point *Z* through which the parallel-circle must be drawn. Unfortunately, no simple emendation suggests itself - the principal difficulty lying in the second mention of "the other head of the compasses" -, but the text is clearly in bad shape and may originally have had the required meaning.

The procedure

$AD = 90$
$BD = \phi$
AZ [on AD] = [Sin] az
$EZ \parallel AG$
EG is the resultant

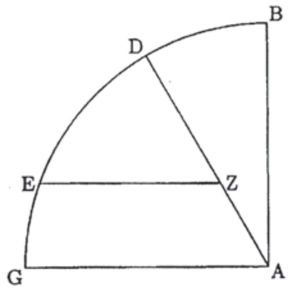

Figure 6

Figure 6 is copied from the diagram in the manuscript. The rule is evidently graduated into ninety unequal parts, so that the graduation at a point Z will make AZ equal to the Sine of that graduation; but otherwise the routine is much as in [1]. Again, Sin EG = Sin az cos ϕ.

[3] To find $\alpha(x)$ for given x and $\delta(x)$.

The procedure

$AD = \delta(x)$
$DE \parallel AB$
$AZ = \bar{\epsilon}$
BZ cuts DE at T
$TH \parallel BG$
Draw BD, cutting TH at K
BK = [Sin] $\alpha(x)$

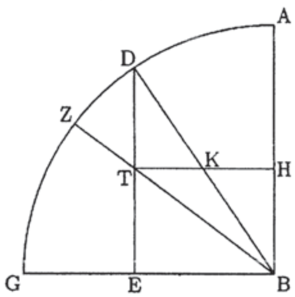

Figure 7

Since $HB = EB \times (\text{Sin } \epsilon \,/\, \text{Cos } \epsilon) = KB \cos \delta$, this yields $BK = \text{Sin } \delta \times$

(Sin ϵ / Cos ϵ) / cos δ. To make this correct, either *AZ* should be made equal to ϵ, not to its complement, or *GZ* must stand for *AZ*. Then the formula is that given in *Almagest* I 16. As in [2] the graduations of the rule (*BD*) ensure that an angle may be read off when the length taken is its sine.

[4] This is the well-known approximate formula for finding the time in seasonal hours from dawn or until dusk[24]. There is no sine-quadrant routine.

[5] To find Sin ½*eq* from δ and ϕ.

The procedure

$AT = \overline{\delta}$
$[TY \parallel AB]$
$GD = \phi$
BD cuts *TY* at *E*
[*ZEH* $\parallel$ *GB*, cutting *BT* at *H*]
BH = [Sin]½*eq*

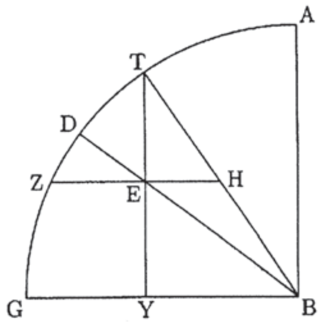

Figure 8

Since $EY = BY \times (\text{Sin}\ \phi\ /\ \text{Cos}\ \phi)$ and $BH = EY\ /\ \cos \overline{\delta}$, this yields $BH = \text{Sin}\ \overline{\delta} \times (\text{Sin}\ \phi\ /\ \text{Cos}\ \phi)\ /\ \cos \overline{\delta}$. Again, either $AT = \delta$, not $\overline{\delta}$, or GT $= \overline{\delta}$. Then the formula conforms to *Almagest* II 3. The routine is identical to that in [3]. Again, the rule is non-uniformly graduated. Possibly "longest" in "longest day" is a mistake.

[6] To find δ from ½*eq* [*r.a.*?], ϕ.

[24] See Girke and King [1988].

The procedure

$AD = \phi$
$GH = \frac{1}{2}eq$; $BE = \text{Sin } \frac{1}{2}eq$
draw BD
$BZ = BE$
$ZT \parallel EH$
$GT =$ total δ

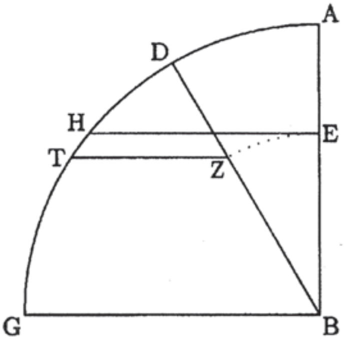

Figure 9 [not in MS]

This yields Sin GT = Sin $\frac{1}{2}eq$ cos ϕ. If $\frac{1}{2}eq$ was mistakenly written for *r.a.*, this is correct and agrees with *Almagest* II 3. If, however, we keep "$\frac{1}{2}eq$", we must alter the procedure. In Figure 9A the following would suffice:

$AD = \phi$
draw BD, DX
$GH = \frac{1}{2}eq$
draw HE
$BZ = BE$
$ZY \parallel EH$ cutting DX at Y
join BY and extend to T
$GT = \delta$

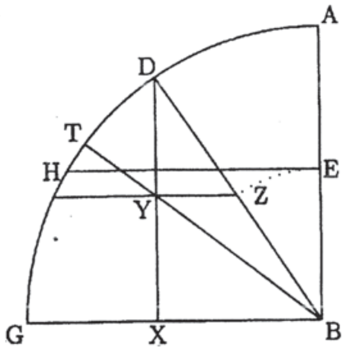

Figure 9A [not in MS]

Except for the position of point *T* this does not contradict the text, but merely adds two lines to the diagram. Either solution to the problem is pure speculation.

[7] To find *r.a.* from δ , ϕ

The procedure

$BG = \delta$
$BD = \overline{\phi}$
[$GZ \parallel BA$, cutting AD at Z]
$AZ =$ [Sin] *r.a.*

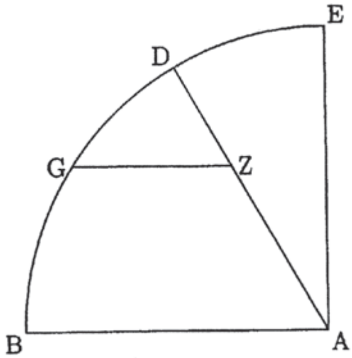

Figure 10

This is equivalent to $AZ = \text{Sin}\ \delta\ /\ \cos\ \phi$, which is correct for *r.a.* Again, an unequally divided rule is used, as in [2].

[8] This is the same as [7], but expressed not as a drawing (albeit with a special rule), but explicitly for an instrument with an alidade and with lines already drawn parallel to one side of the quadrant.

The procedure

[See Figure 10] $BD = \overline{\phi}$; $BG = \delta$; look along parallel to *BA* through *G* to find *Z*.

"the sixty degrees" (or "parts") in the penultimate sentence is probably an insertion, since we want to read off the result in degrees, as in [2] etc.; and in this case "the 90 degrees" would be appropriate.

Text II (*BN ar. 2457, f.* 95*r*)

بسم الله الرحمان الرحيم

إذا أردت أن تعرف الساعات المستوية من ظهر الأصطرلاب فاعرف نصف قوس نهارك وضع حرف العضادة على خط وسط السماء وانظر أي جزء قطع حرف العضادة من الجيب المخرج من نقطة قوس النهار وهو القوس الذي ما بين ص وبين أي جزء وقع من أجزاء الارتفاع وهو في ناحية الجنوب ثم عد من القطب بمقدار ما بين حرف العضادة وبين ما وقع الجيب عليها إلى أي جزء وقع من أجزاء العضادة فعلّم هناك علامة ثم حرك العضادة حتى وقع الجيب المعكوس من غاية الارتفاع وهو الخط المخرج من نقطة غاية ارتفاع ذلك اليوم نحو المشرق على موضع العلامة وانظر الخط الموازي له من نقطة ارتفاع ذلك الوقت على أي موضع وقع من العضادة وهذه القسي جميعا ينبغي أن يكون ابتداؤها من ص إلى خلاف توالي درجات الارتفاع ثم انظر العدد الذي بين العلامتين من العضادة ثم عد[25] بمقدار ذلك العدد من حرف العضادة نحو القطب فحيث بلغ فعلّم هناك علامة ثم ضع العضادة على خط نصف النهار وانظر إلى الدرجات[26] التي وقع جيبها[27] على تلك العلامة فعد من موضع ص إلى تلك النقطة فما كان فهو درجات مطلعية من وقت نصف النهار إلى وقت ما دار من الفلك، وذلك ما أردنا.

[§1] When you want to determine the equal hours from the back of the astrolabe, determine half the arc of your day and put the edge of the alidade on the meridian line and see which degree cuts the edge of the alidade from the sine drawn from the point of the arc of the day, which is the arc between 90 and whichever of the degrees of altitude it falls [on] - and it is on the south side.

[25] عد: عدد في م.

[26] الدرجات: درجات في م.

[27] جيبها: جيبه في م.

[§2] Then count from the pole, in the quantity of what is between the edge of the alidade and what the sine falls on, to whichever of the degrees of the alidade it falls [on]. Make a mark there.

[§3] Then move the alidade until the versed sine of the maximum altitude (which is the line drawn from the point of the maximum altitude of that day) towards the east falls at the position of the mark. Look at the line parallel to it from the point of the altitude of that time, on which position on the alidade it falls [and make a mark]. These arcs together: their beginning must be at 90, in direction against the sequence of degrees of altitude.

[§4] Then look at the amount [*ᶜadad*] which is between the two marks on the alidade. Then count in the quantity of that amount on [*min*] the edge of the alidade towards the pole: where it reaches, make a mark there.

[§5] Then put the alidade on the meridian line and look at the degrees whose sine falls on that mark. Count from the 90-position to that point: what it is is the ascensional degrees from the time of midday to the time of what has rotated from the Sphere. That is what we wanted.

Commentary

For convenience of reference the translation has been divided into numbered paragraphs. Let d be half the arc of day, t the time since (or until) midday, h the altitude of the Sun and H its altitude at midday. The operations in §1, §3 and §5 are illustrated in Figure 11. §2 and §4 are about transferring distances on the alidade to its lower or upper part. When on the meridian the alidade is shown separated from the quadrant. All marks are made on the alidade.

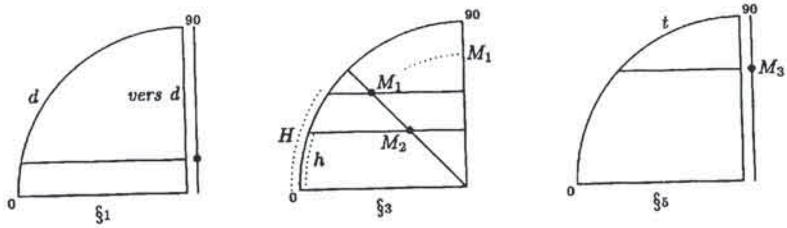

Figure 11

§1-§2 Put the alidade on the meridian and, with the help of the appropriate parallel-line, find *vers d*. Put a mark (M_1, say) so that between it and the center *vers d* is measured.

§3 Move the alidade so that the mark falls on the parallel for altitude H. Make another mark (M_2) where it crosses the altitude line for h.

§4 Transfer the distance between M_1 and M_2 to the upper part of the alidade, so that one end of the distance is the tip of the alidade. Put a mark (M_3) at the other end.

§5 With the alidade back at the meridian, read off the arc whose versed sine is this distance: this is t (in degrees).

The text is not altogether clear. The worst doubt about the above interpretation concerns §3. Its last sentence and the term "versed sine" seem to imply that the two altitudes should be measured from the 90-point, but this would not produce the right result. It may be that the last sentence is displaced or has been added. We note in passing that "end" or "tip" is probably the meaning of *ḥarf* ("edge") in §2 and §4, but not in §1.

If we label the upper end of the alidade C and the lower end A, as in Figures 1 and 2, we may express the procedure in trigonometrical terms:
§1 and §2 $AM_1 = \text{vers } d$

§3 $AM_2 : AM_1 = \text{Sin } h : \text{Sin } H$
§4 and §5 $\text{vers } t = AM_1 - AM_2$

Thus the procedure is equivalent to the Indian formula[28]

$\text{vers } t = \text{vers } d - (\text{Sin } h / \text{Sin } H) \text{ vers } d$

This formula was well known in the earlier phase of Arabic astronomy, e.g. to Ḥabash, al-Khwārizmī, al-Battānī (d. 929) and Abū'l-Wafā' (d. 998)[29].

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[28] See M.-Th. Debarnot, p. 37. Here the formula is called I3.

[29] Ibid., pp. 60, 45n76, 40, 4n5, respectively.

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